Part B-1

1 Excellence

1.1 Quality and pertinence of the project's research and innovation objectives

The design of formal methods to express and reason about **recursion** is one of the main challenges of contemporary logic. This theoretical challenge finds several fundamental applications in computer science. Consider, e.g., the validation of software, as relevant to *formal verification*. In this area, recursively defined notions are needed to reason about iterative behaviours of programs. For instance, let ϕ be the desirable property "non-authorised users do not have access to classified information". Recursion is needed to express the following statement, regarding a specific program π:

Successfully executing π *any finite number of times* results in a state where ϕ holds. (1)

As a second prominent example, in *knowledge representation* recursion is required to express *common knowledge*, that is, the following infinite sentence, where "everybody" denotes all students is a classroom, ϕ stands for "the assignment is due tomorrow" and the professor has just announced that ϕ:

Everybody knows that ϕ and everybody knows that everybody knows that ϕ and ... (2)

Formal tools to study recursion include modal fixpoint logics and proof systems. **Modal fixpoint logics**, expressing recursively defined notions, are currently employed in formal verification and knowledge representation. Much as in basic modal logic, the semantics of modal fixpoint logics can be enriched with additional properties, called **frame conditions**, to capture specific attitudes, e.g. knowledge. Frame conditions give rise to rich and expressive families of modal fixpoint logics.

Proof theory, the discipline studying **proof systems**, plays a fundamental role in the analysis of logical systems. A proof system is defined by a set of axioms and inference rules, used to check the validity of a formula by constructing a relevant *derivation*, that is, a proof tree composed from the axioms and rules of the proof system. Despite their wide range of application, the proof theory of modal fixpoint logics is currently underdeveloped. More specifically, the proof theory of modal fixpoint logics with frame conditions is much less advanced than the proof theory of modal or intuitionistic logics, due to non-trivial combinatorial challenges.

CYDER will develop the proof theory of modal fixpoint logics, with the following **key objective**:

Define ***proof systems*** *for* ***modal fixpoint logics*** *with* ***frame conditions***.

Towards this challenging aim, CYDER will define two new proof systems: **labelled** and **nested cyclic proofs**, introducing a novel approach inspired by the methods defined for:

- **Cyclic proofs**, which treat operators expressing forms of recursion. They are a powerful formalism to deal with infinite proof trees using finite representations.
- **Labelled** and **nested sequent calculi**, which capture families of modal logics with frame conditions.

CYDER will introduce an original proof-theoretical approach to modal fixpoint logics with frame conditions, significantly advancing the state of the art by modularly defining labelled and nested cyclic proofs for families of these logics. At present, a general mathematical and proof-theoretical understanding of modal fixpoint logics with frame conditions is missing.

CYDER will initially focus on epistemic and temporal modal fixpoint logics, and then extend the investigation to more complex systems: more specifically, it will introduce cyclic proofs for **Intuitionistic modal logics** (IML) and **Dynamic Epistemic Logics** (DEL), both extended with recursive modalities. These logics have significant applications in formal verification and knowledge representation, but have been studied only from a model-theoretical viewpoint.

CYDER objective is ambitious, as it represents the first comprehensive study of modal fixpoint logics, treating the complex interaction between recursive modal operators and frame conditions. While challenging, the objective is feasible, since it relies on well-established methods from the literature of modal and fixpoint logics. In this perspective, the applicant's expertise on the proof theory of modal logics and the supervisors' extensive knowledge of modal fixpoint logics and cyclic proofs will offer an additional guarantee for the success of this ambitious program.

Upon completion, CYDER will provide a general proof-theoretical framework for classical and intuitionistic modal fixpoint logics with frame conditions. This will boost the exploitation of these logics in representing temporal and epistemic properties, offering highly-needed tools in key areas of computer science, such as formal verification.

State of the art and contributions of the project

Modal logics Modal logics enrich the language of classical propositional logics by *modal operators*. A few examples of modal formulas and their interpretations are listed below:

Modal fml.	*Interpretation*	*Logic*
$\Box\phi$	"ϕ is necessary"	Alethic logic
$\bigcirc\phi$	"At the next moment of time, ϕ will hold."	Temporal logic
$K_a\phi$	"Agent a knows that ϕ."	Epistemic logic
$[\pi]\phi$	"Successfully executing program π results in a state where ϕ holds."	Dynamic logic

The modal operators in the last two formulas are indexed by labels for agents (a) and for programs (π). *Mono-modal logics* employ only non-indexed modalities, while *multi-modal logics* include indexed ones.

The **semantics** of multi-modal logics is defined in terms of *Labelled Transition Systems* (LTS). An LTS is composed of a *frame*, which is a directed graph with labelled edges, and a *valuation*, i.e. a function assigning propositional formulas to the nodes of the graph (see Fig. 1). The semantics of the basic multi-modal logic, mK, is the class of arbitrary LTS. The semantics of mono-modal logics is defined by means of LTS with unlabelled edges, which are also called *Kripke models*[1].

A natural request when modelling the behaviour of modalities is that LTS **frames** satisfy specific **conditions**. For instance, when modelling knowledge, edges with the same label are usually required to form an equivalence relation. Frame conditions allow to identify a number of logics. The most well-known mono-modal logics are represented in the S5-*cube* in Fig. 1.

Labelled[2] and **nested**[3] sequent calculi are proof systems modularly capturing the mono-modal logics in the S5-cube. Both calculi have also been defined for multi-modal logics. While sequents for classical propositional logic are *multisets* of formulas, sequents for modal logics are characterised by more complex data structures, reflecting the rich semantics of their logics. Labelled sequents encode *graphs* of formulas by adding semantic information within the language of the calculi. Nested sequents are *trees* of formulas, constructed by enriching the structure of sequents.

⇒ CYDER will exploit the modular proof-theoretical methods for modal logics to introduce labelled and nested sequents into the richer framework of modal fixpoint logics.

Modal fixpoint logics Recursion can be formally expressed by two kinds of operators:

- *Recursive modalities*, expressing inductively defined notions, as in statements (1) and (2) above. Recursive modalities can be added to temporal, dynamic and epistemic logics.
- *Explicit fixpoint operators*, μ and ν, representing the *least* and *greatest* fixpoints of functions.

The *modal μ-calculus* is the logic resulting from adding explicit fixpoints to multi-modal logic mK. Logics with explicit fixpoints are more expressive than logics with recursive modalities, that can be defined in terms of μ or ν. The *alternation-free fragment* of the modal μ-calculus contains only μ-calculus formulas in which μ and ν do not interact. The logic is still expressive enough to capture most temporal and dynamic operators[4]. With the name **modal fixpoint logics** we denote both the systems with recursive modalities and those with explicit fixpoints.

As in basic modal logic, LTSs for modal fixpoint logics can be enriched with **frame conditions**. Despite their applications in key areas of computer science, a comprehensive model-theoretic or proof-theoretic

[1] P. Blackburn, M. d. Rijke, and Y. Venema, *Modal Logic*. Cambridge University Press, 2001,
[2] S. Negri, "Proof analysis in modal logic," *Journal of Philosophical Logic*, vol. 34, no. 5-6, 2005.
[3] K. Brünnler, "Deep sequent systems for modal logic," *Archive for Mathematical Logic*, vol. 48, no. 6, 2009.
[4] J. Marti and Y. Venema, "A focus system for the alternation-free mu-calculus," in *TABLEAUX 2021*, Springer, 2021,

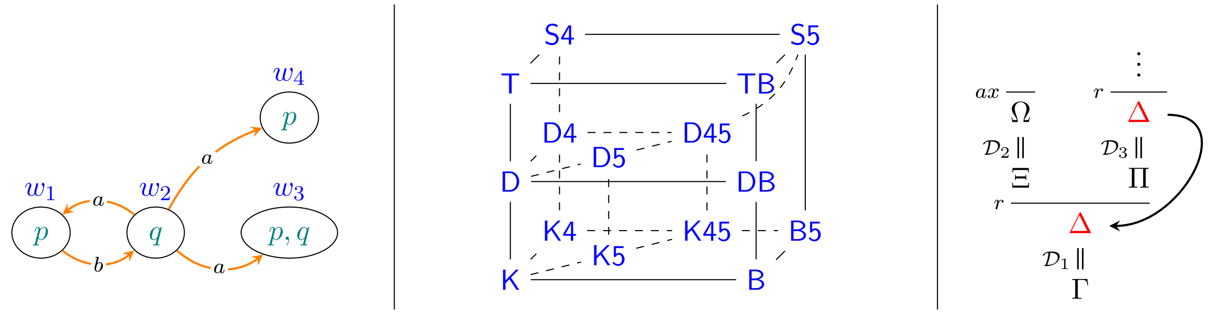


Figure 1: **Left:** a Labelled Transition System. Formula $[a]\,p$ is valid at world w_2, because p appears in all the worlds a-accessible from w_2. **Center:** the S5-cube of mono-modal logics. K is the basic logic; the letters denote the following frame conditions: D *seriality*; T *reflexivity*; 4 *transitivity*; B *symmetry* and 5 *euclideaness*. **Right:** a cyclic pre-proof. Sequent Δ and the subderivation above it would keep repeating along the branch.

study of modal fixpoint logics with frame conditions has not been developed so far. It is well-known[1] that adding recursive modalities to systems of modal logics might result in logics which are *highly undecidable*, meaning that the valid formulas of the logic might not be recursively enumerable. High undecidability would prevent the definition of a finite set of rules for the logic. However, a taxonomy of the decidable and undecidable logics resulting from the interaction between frame conditions and recursive modalities is currently missing from the literature.

⇒ By conducting a proof-theoretical study of modal fixpoint logics with frame conditions, CYDER will offer the first comprehensive analysis of these logics, contributing to draw the boundaries of decidability.

Cyclic proofs Strategies to treat recursion are needed in proof systems for modal fixpoint logics. Several methods have been proposed in the literature. Among them, CYDER will focus on cyclic proofs. Cyclic proofs are a special kind of **non-wellfounded proofs**, which are infinitary proofs treating recursion by means of rules which "unfold" the recursive operators infinitely often. In this formalism, proofs are constructed using only rules with a finite number of premises, but branches can be infinitely long. A **cyclic pre-proof** is a non-wellfounded proof whose infinite branches are regular, or equivalently, it is a finitary proof equipped with a back-link. Cyclic pre-proofs can be represented as finite (cyclic) graphs, as shown on the right of Fig. 1. Then, a **cyclic proof** is a cyclic pre-proof whose cyclic branches satisfy a *progress condition*, to guarantee a specific formula in the cyclic branch which gets unfolded infinitely often by a fixpoint rule. Cyclic proofs have an explicit connection with automata and game theory, thus enriching proof theory with powerful and well-established computational tools[5].

⇒ CYDER will introduce cyclic proof systems for modal fixpoint logics on the basis of labelled and nested sequents. Thus, CYDER will extend the scope of cyclic proofs with strategies to capture frame conditions.

Cyclic proofs for modal fixpoint logics Niwiński and Walukiewicz introduced cyclic proofs for the modal μ-calculus in terms of tableau games[6]. Since then, cyclic proof theory was developed for several logics. Sequent calculi based on cyclic proofs have found applications in modal logics[7], in first order logics extended with inductively defined predicates[8], and in linear logic[9]. A recent breakthrough in cyclic proof theory was obtained by Afshari and Leigh[10], who proved completeness of Kozen's finitary proof system for the modal μ-calculus[11], using purely proof-theoretical methods.

Most cyclic proof systems for modal fixpoint logics defined in the literature use classical sequents, since they are defined for simple systems of modal fixpoint logics, i.e., *without* frame conditions. To develop the proof theory of modal fixpoint logics with frame conditions, CYDER will introduce original

[5]E. Gradel and W. Thomas, *Automata, logics, and infinite games: A Guide to Current Research*. Springer, 2002.
[6]D. Niwiński and I. Walukiewicz, "Games for the mu-calculus," *Theoretical Computer Science*, vol. 163, no. 1, 1996.
[7]K. Brünnler and M. Lange, "Cut-free sequent systems for temporal logic," *The Journal of Logic and Algebraic Programming*, vol. 76, no. 2, 2008.
[8]J. Brotherston and A. Simpson, "Sequent calculi for induction and infinite descent," *Journal of Logic and Computation*, vol. 21, no. 6, 2011.
[9]D. Baelde, A. Doumane, and A. Saurin, "Infinitary proof theory: The multiplicative additive case," in *CSL 2016*, 2016.
[10]B. Afshari and G. E. Leigh, "Cut-free completeness for modal mu-calculus," in *LICS 2017*, IEEE, 2017,
[11]D. Kozen, "Results on the propositional mu-calculus," *Theoretical computer science*, vol. 27, no. 3, 1983.

proof systems, merging cyclic proofs and methods to define labelled and nested sequents. In this direction, cyclic non-classical sequents have been introduced for Kleene algebra[12] and for some mono-modal logics with a recursive modality[13], and cyclic labelled proofs have been defined for *Propositional Dynamic Logic*[14]. However, a comprehensive proof-theoretic analysis of logics with recursive modalities and frame conditions has not been developed so far.

⇒ As demonstrated by the recent results mentioned above, cyclic proofs are an increasingly important topic for the proof-theoretical community. CYDER will contribute to the field by defining proof systems for several logics, including mono-modal logics from the S5-cube extended with recursive modalities, multi-modal epistemic logic S5 with common knowledge and the alternation-free fragment of the modal μ-calculus with frame conditions.

IML with recursive modalities Intuitionistic propositional logic formalises *constructive reasoning* by rejecting classical principles, such as the *excluded middle*, $\phi \vee \neg\phi$. IML are defined adding mono-modal operators to intuitionistic, rather than classical, propositional logic. Situations in which only *partial information* is available can be formalised with IML: for instance, starting from a state x, one might not yet know whether process π can be executed so that, in an intuitionistic framework, neither $[\pi]\phi$ nor $\neg[\pi]\phi$ hold at x[15]. Recursive modalities added to IML express forms of recursion in a constructive setting. While some model-theoretic analysis of IML with recursive modalities have been proposed[16], the proof theory of these logics has not yet been explored.

⇒ CYDER will introduce the first proof systems for IML extended with recursive modalities, thus paving the way to further applications of this expressive formalism in formal verification.

DEL with recursive modalities DEL are extensions of epistemic multi-modal logics with *dynamic* modal operators, i.e., operators of the form $[\pi]$, where π describes a change in the model. CYDER will focus on two DEL systems, both suitable to be expanded with recursive modalities.

- The *Logic of Doxastic Upgrades*[17] represents changes in an agent beliefs. Models for this logic are LTSs whose edges form a *total preorder*, representing beliefs of agents. Dynamic modalities describe *upgrades* of agents beliefs, and recursive modalities allow to reason about iterated upgrades.
- The *Epistemic Logic of Threshold-Limited Influence*[18] models the diffusion of a behaviour through a network of agents, where an agent adopts a new behaviour only if a certain number of her neighbourhoods has already adopted it. LTSs for this logic have *irreflexive*, *symmetric* and *serial* frames, and represent a network of agents. Dynamic modalities describe the state of the network at the next moment in time, i.e., when new agents may have adopted the behaviour, and recursive modalities capture the result of iterating the process.

⇒ CYDER will introduce the first proof systems for the two logics above, offering the first proof-theoretical treatment of DEL with recursive modalities.

1.2 Soundness of the proposed methodology

The research objectives of CYDER are organised into three technical work packages, illustrated below. The core methodology of CYDER will be mostly developed in the course of WP1, and extended to IML and DEL in the course of WP2 and WP3. The two supervisors of CYDER, Prof. Venema and Dr. Afshari, both from the *Institute for Logic, Language and Computation* (ILLC) of the University of Amsterdam

[12] A. Das and D. Pous, "A cut-free cyclic proof system for Kleene algebra," in *TABLEAUX 2017*, Springer, 2017.
[13] J. Rooduijn, "Cyclic hypersequent calculi for some modal logics with the master modality," in *TABLEAUX 2021*, Springer, 2021.
[14] S. Docherty and R. Rowe, "A non-wellfounded, labelled proof system for propositional dynamic logic," in *TABLEAUX 2019*, Springer, 2019.
[15] K. Y. Ahn, R. Horne, and A. Tiu, "A characterisation of open bisimilarity using an intuitionistic modal logic," *Logical Methods in Computer Science*, vol. 17, no. 3, 2021.
[16] J. Degen and J. Werner, "Towards intuitionistic dynamic logic," *Logic and logical philosophy*, vol. 15, no. 4, 2006.
[17] A. Baltag and S. Smets, "Keep changing your beliefs, aiming for the truth," *Erkenntnis*, vol. 75, no. 2, 2011.
[18] A. Baltag, Z. Christoff, R. K. Rendsvig, *et al.*, "Dynamic epistemic logics of diffusion and prediction in social networks," *Studia Logica*, vol. 107, no. 3, 2019.

(UvA), are world-experts in modal fixpoint logics and their proof theory. Interactions with them will be crucial to deliver the objectives of CYDER. Moreover, for WP3 I foresee to extend the collaboration to two leading experts in DEL: Prof. Smets and Dr. Baltag, both from ILLC.

- WP1 **Cyclic proofs for modal logics with recursive modalities and frame conditions.** I will first consider mono-modal logics and frame conditions from the S5-cube, and simple non-indexed recursive modalities such as the *master modality*, which is the reflexive and transitive closure of $\Box$. I will then extend the investigation to multi-modal epistemic logics with relevant frame conditions and to the *alternation-free fragment* of the modal μ-calculus with frame conditions.

- WP2 **Cyclic proofs for IML with recursive modalities.** I will define cyclic nested sequents for families of IML extended with simple non-indexed recursive modalities. I will then define a cyclic proof system for intuitionistic PDL.

- WP3 **Cyclic proofs for DEL with recursive modalities.** I will define cyclic labelled proof systems for two multi-modal systems with frame conditions: the *Logic of Doxastic Upgrades* and the *Epistemic Logic of Threshold-Limited Influence*, both extended with indexed recursive modalities. The semantics of both logics display specific frame conditions.

Methodology The methodology employed to deliver the research objectives of CYDER combines the well-established proof-theoretical methods to define analytic sequent calculi for families of modal logics and the strategies to construct cyclic proofs, up to now mostly employed for modal fixpoint logics defined on the basic system of modal logic (K or mK). I will initially consider mono-modal logics and frame conditions from the S5-cube, and will then extend the investigation to multi-modal logics with frame conditions: multi-modal epistemic logics and the alternation-free fragment of the modal μ-calculus (WP1) and systems of DEL (WP3). I will also extend CYDER methods to the intuitionistic framework (WP2).

CYDER main **challenge** is the identification of the correct notion of *pre-proof* within labelled and nested frameworks. Nested and labelled sequents encode tree- or graph-structures which might "grow" indefinitely during proof search along a branch, preventing the identification of a loop, and consequently the drawing of a back-link, along the branch. For WP1, I will tackle this issue using the termination strategies defined for labelled and nested sequents for modal logic. These strategies, which stop proof search when a labelled or nested sequent grows by repeating one of its sub-graphs, offer precious insights on when an infinite branch created by unfolding a recursive operator can be considered cyclic. I will then generalise the methods developed in WP1 to treat the case of the alternation-free modal μ-calculus. To obtain the results described in WP2 I will rely on nested sequents for IML defined in the literature[19] and on the methods developed in the course of WP1. Regarding WP3, display calculi for some systems of DEL *without* recursive modalities have been proposed, employing a quite complex formalism[20]. CYDER plans to use the labelled approach[21] as a blueprint to develop modular calculi for systems of DEL, suitable to include recursive modalities. Cyclic proofs will be defined by exploiting the methods defined in WP1 to treat multi-modal logics. The objectives in WP2 and WP3 can also be developed independently from WP1, as they treat different systems of logics.

Interdisciplinarity The research program of CYDER requires a multi-disciplinary effort, bringing together expertise from several disciplines. CYDER relies on proof-theoretic techniques to define analytic proof systems (**logic**) and on cyclic proofs, which allow to treat recursion (**computer science**). Moreover, the logics express fixpoints of functions (**mathematics**) and find applications in representing the epistemic attitudes of agents (**philosophy**). The integration of techniques and perspectives from different disciplines is the characterising trait of the ILLC, the host institution, which brings together experts in the above fields, thus providing CYDER with the ideal environment to reach its goals.

Gender Not applicable, because the research of CYDER is of an abstract and theoretical nature.

[19] L. Straßburger, "Cut elimination in nested sequents for intuitionistic modal logics," in *FoSSaCS'13*, Springer, 2013.

[20] S. Frittella, G. Greco, A. Kurz, *et al.*, "Multi-type display calculus for dynamic epistemic logic," *Journal of Logic and Computation*, vol. 26, no. 6, 2016.

[21] M. Girlando, S. Negri, N. Olivetti, *et al.*, "Conditional beliefs: From neighbourhood semantics to sequent calculus," *The Review of Symbolic Logic*, pp. 1–44, 2018.

Open science and research data management All the results of CYDER will be made available as arXiv pre-prints, with the aim of encouraging informal evaluation from the scientific community. Moreover, I will (co-)organise a workshop to communicate results and encourage feedback from other researchers (Sec. 2.2). The project deliverables will be published in the proceedings of high-rated and peer-reviewed conferences and in scientific journals. Proceeding publications will be made available on arXiv, and journal contributions will be published in Open Access format, for which funding is foreseen. CYDER does not envisage the collection of any kind of data, nor the implementation of programs.

1.3 Quality of the supervision, training and of the two-way transfer of knowledge between the researcher and the host

Quality of the supervision The project will be supervised by Prof. Venema and Dr. Afshari at the *Institute for Logic, Language and Computation* (ILLC) of the University of Amsterdam (UvA). Prof. Venema, main supervisor of CYDER, is a leading expert in modal, coalgebraic and fixpoint logics. He has an excellent record of publications, comprising 4 (co-)authored books, among which the reference text *Modal logic*, and more than 90 publications in journals and peer-reviewed conference proceedings. His recent results on cyclic and annotated proofs systems for the modal μ-calculus are of direct interest to CYDER[4], and his expertise in the model-theory of modal fixpoint logics, and in game and automata theory, will be essential for CYDER success. Prof. Venema has an excellent track record in mentoring young researchers: he supervised more than 10 Ph.D. students and 8 postodcs, and served as primary supervisor of 3 Marie Skłodowska Curie Individual Fellowships.

Dr. Afshari is a world expert in the proof theory of modal fixpoint logics. Among her publications, of special interest are the breakthrough proof of completeness for Kozen's axiomatization[10], and the definition of an infinitary proof system for the full modal μ-calculus[22]. Dr. Afshari was awarded a MacGillavry Fellowship at UvA, a grant for outstanding researchers, and she is currently in charge of the relevant UvA research project. Moreover, Dr. Afshari is PI of a Swedish Research Council Starting Grant and of a project funded by the Dutch Research Council (NWO). These research grants investigate different aspects of modal fixpoint logics.

Prof. Venema and Dr. Afshari lead the project *Proof Systems for Modal Fixpoint Logics*, whose research interests closely match CYDER objectives[23]. The project merges an NWO TOP grant awarded to Prof. Venema and Dr. Afshari's UvA project. In the larger frame of this project, CYDER specifically focuses on cyclic proofs and on modal fixpoint logics with frame conditions. My participation to the project meetings, scheduled twice per month, will guarantee the smooth integration of CYDER in the project.

Host-to-applicant transfer of knowledge The training program of CYDER will strengthen my scientific and professional competences. Regarding **scientific training**, I will deepen my proof-theoretical and model-theoretical understanding of fixpoint operators through weekly meetings with Prof. Venema. Interactions with Dr. Afshari will strengthen my knowledge of proof systems for the modal μ-calculus, and of the relations between them. Through exchanges with Prof. Smets and Dr. Baltag, leading researches in the field, I will deepen my knowledge of DEL and their semantics. I will fully exploit the outstanding learning opportunities offered by ILLC, attending the following Master courses: *Logic, Games and Automata*, taught by Dr. Afshari[24], and *Dynamic Epistemic Logic*, taught by Dr. Baltag[25].

Being in charge of a two-year fellowship will in itself be an opportunity to reinforce a number of **professional skills**, most notably **organisational competences**. Moreover, the UvA offers several courses and workshops, including support services for Dutch and European grants[26]. I will take advantage of this opportunity to improve my **grant writing** skills, also attending the workshops dedicated to national and ERC Starting Grants. I will follow the *cursus* to obtain the *Basis Kwalificatie Onderwijs* (BKO), the qualification required to teach at Dutch Universities[27]. This implies attending courses on various aspects of **teaching** and mentoring, as well as taking part in a practical training with actual academic lessons. To

[22] B. Afshari, G. Jäger, and G. E. Leigh, "An infinitary treatment of full mu-calculus," in *WoLLIC 2019*, Springer, 2019.
[23] https://sites.google.com/view/psmfl/home
[24] https://studiegids.uva.nl/xmlpages/page/2021-2022-en/search-course/course/90217
[25] https://studiegids.uva.nl/xmlpages/page/2021-2022-en/search-course/course/88985
[26] https://grant-support.uva.nl/news-workshops--events/workshops--events/events.html
[27] https://tlc.uva.nl/en/article/university-teaching-qualification-bko/

increase my **mentoring** skills I will propose projects related to the topics of CYDER to the students of the Master of Logic[28]. I plan to attend **diversity training** initiatives organised by the *Chief Diversity Officer team* of the UvA, e.g, the workshop on *Implicit Bias and Open Communication*[29].

Applicant-to-host transfer of knowledge I will bring to ILLC my knowledge in structural proof theory and, specifically, my expertise on analytic calculi for modal logics and intuitionistic modal logics. I will share my knowledge through one-to-one interactions with ILLC members and students, by speaking at seminars, by supervising Master projects and by (co-)organising a workshop at UvA on CYDER topics.

1.4 Quality and appropriateness of the researcher's professional experience, competences and skills

I am an active researcher in the proof theory of modal logics, as demonstrated by my strong publication record. My 11 published contributions include 2 journal papers and 9 peer-reviewed papers published in the proceedings of top-ranking conferences in my research area, including AiML and TABLEAUX. This demonstrates my skills as a qualified researcher, able to effectively disseminate the research outputs. My publications, co-authored with researchers coming from different fields, testify my attitude towards multi-disciplinary research, and my CV demonstrates my ability to smoothly integrate in labs located in different countries.

My formation is highly interdisciplinary: after a Master degree in logic and philosophy at the University of Florence, I obtained a double Ph.D. degree, in computer science and philosophy. I was jointly supervised by Prof. Olivetti (University of Aix-Marseille) a leading expert in the proof theory of modal and non-classical logics, and by Prof. Negri (University of Helisnki) author of pioneering works on labelled proof systems. During my Ph.D., I studied the proof theory of conditional and epistemic logics, producing influential results such as the introduction of the first sequent calculi for preferential conditional logics[30], and the definition of nested sequents for the *Logic of Conditional Belief*, an expressive multi-modal logic[31]. The thesis is published[32], and obtained the **Best Thesis award** of the Univeristy of Aix-Marseille. After the Ph.D., I was granted a postdoctoral position at Inria Saclay-LIX, where I deepened my knowledge of nested sequents for IML[33]. Since March 2021 I am a Reserach Fellow at the University of Birmingham, within the UKRI project *Structure vs. Invariants in Proofs* led by Dr. Das[34]. The project aims at studying cyclic proofs using proof-theoretical tools, and, thanks to it, I am building my expertise in cyclic proof theory, essential to deliver the objectives of CYDER.

2 Impact

2.1 Credibility of the measures to enhance the career perspectives and employability of the researcher and contribution to his/her skills development

My main professional aim is to obtain a tenured position in academia. The portfolio of technical and transverse skills offered by CYDER, as described above, will form a solid basis towards this aim. At the beginning of the action, a *Personal Career Development Plan* (PCDP) will be agreed with Prof. Venema and Dr. Afshari, with the aim of defining specific training needs and goals. The PCDP will be revised and updated twice per year, as to fully exploit CYDER opportunities towards my professional fulfilment.

The **scientific expertise** that I will gain in the course of the project will allow me to pursue impactful lines of work, e.g. by applying CYDER proof-theoretical methods to other logics. This will establish my active role in the proof-theoretical community and increase my visibility, thus boosting my chances to

[28] https://msclogic.illc.uva.nl/current-students/courses/projects/

[29] https://www.uva.nl/en/about-the-uva/about-the-university/diversity-and-inclusion.html

[30] M. Girlando, S. Negri, and N. Olivetti, "Uniform labelled calculi for preferential conditional logics based on neighbourhood semantics," *Journal of Logic and Computation*, vol. 31, no. 3, 2021.

[31] M. Girlando, B. Lellmann, and N. Olivetti, "Nested sequents for the logic of conditional belief," in *JELIA 2019*, Springer, 2019.

[32] M. Girlando, "On the proof theory of conditional logics," Ph.D. dissertation, University of Helsinki, 2019.

[33] L. Straßburger, "Cut elimination in nested sequents for intuitionistic modal logics," in *FoSSaCS'13*, Springer, 2013.

[34] https://gtr.ukri.org/projects?ref=MR%2FS035540%2F1#:~:text='Structure%20vs.,known%20as%20Hilbert's%2024th%20problem.

obtain an academic position. My **publication record** will be significantly increased by the research output of CYDER (2 journal papers and 3 conference papers), granting me a favourable evaluation in recruitment processes. Obtaining the Marie Skłodowska Curie fellowship will demonstrate my capacity of **raising funds**, a quality well sought-after for tenured positions. The **organisational and teaching skills** developed during the action will strengthen key features of my CV, and my engagement in **outreach activities** (Sec. 2.2), will distinguish me from other applicants. Moreover, the BKO qualification will give me the opportunity to apply to Dutch tenured positions and the attendance to the grant writing workshops will put me in a favourable position to obtain qualified research grants (NWO or ERC Starting Grant).

While my primary goal is obtaining an academic position, the competences and skills developed during the action are also a valuable bonus to obtain qualified positions in non-academic environments, specifically in the fields of software research and development, implementation of ontologies and formal approaches to artificial intelligence.

2.2 Suitability and quality of the measures to maximise expected outcomes and impacts, as set out in the dissemination and exploitation plan, including communication activities

The achievements of CYDER will be presented at international workshops, e.g. WoLLIC, published in proceedings of peer-reviewed conferences, such as LICS, TABLEAUX, AiML, TARK, LORI, and in high-impact scientific journals, such as the *Review of Symbolic Logic* and *Logical Methods in Computer Science*.

CYDER foresees a dissemination and exploitation plan targeting various audiences. Actions directed to the **scientific community** include a **workshop** on proof methods for cyclic proof systems, that I will (co-)organise at month 12, with the aim of communicating CYDER's results and bringing together the growing community of researchers interested in cyclic proofs. To promote CYDER's objectives, I will propose **seminars** to research groups throughout Europe and, thanks to the new technologies (e.g., Zoom) throughout the world. Moreover, I will propose a **course** introducing cyclic proof theory and directed to Ph.D. and Master students at the *European Summer School in Logic, Language and Information* (ESSLLI). Even though I attended several summer schools, I currently do not have experience in teaching intensive courses and, to make up for this, I will seek the help of a more experienced colleague.

Regarding actions targeting **high-school students and teachers**, I will participate to the organisation of ILLC MasterClasses, both those directed at teachers and those targeting students[35]. I also plan to propose internship for students modelled on the *Hippocampes*, to which I took part as tutor in 2017 and 2018[36]. I intend to engage in outreach activities aimed to the **general public** by participating to ILLC Open Days and other outreach events, including the European Researchers' Nights. Lastly, updates regarding CYDER's progress and activities will be shared through a webpage and on social media (Twitter).

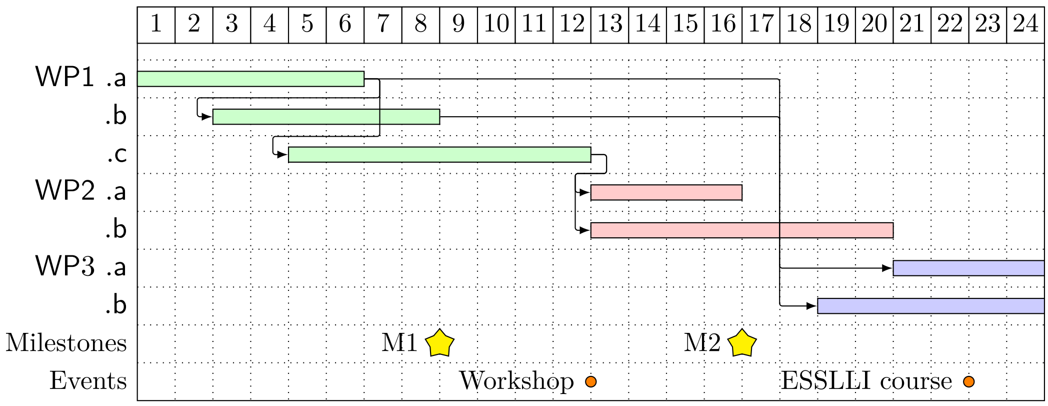
Intellectual property CYDER does not foresee any commercial exploitation, and all publications will be openly accessible. In the unlikely event of intellectual property issues, the ILLC office will offer support.

Target group	Message	Activities	Tools / *Channels*
Scientific community	Communicate CYDER's results	Workshop	Informal proceedings
		Seminars	Pre-prints, slides
Master, Ph.D. students	Introduction to cyclic proofs	ESSLLI course	Course material
High-school students and teachers	Introduction to mathematical and logical concepts and methods	ILLC MasterClass	Lessons
		Internships	Research problems
General public	Familiarisation with logical reasoning and promotion of CYDER	ILLC Open days	DEL puzzles
		Outreach events	Short accessible talks
		Website, Twitter	Blog posts

2.3 The magnitude and importance of the project's contribution to the expected scientific, societal and economic impacts

Expected scientific impact The multi-disciplinary CYDER action will have a significant impact in different fields. The immediate output will concern the proof-theoretical community, but important repercussions

[35] https://events.illc.uva.nl/MasterClass/
[36] https://hippocampe.irem.univ-mrs.fr/



Tasks	*Description*	*Deliv.*	*Month*
• WP1.a	Cyclic labelled sequents for mono-modal logic + FC + RM	D1 - PP	6
• WP1.b	Cyclic labelled proof systems for multi-modal S5 + common knowledge	D2 - PP	8
• WP1.c	Cyclic nested sequents for alternation-free modal μ-calculus + FC	D3 - JP	12
• WP2.a	Cyclic nested sequents for IML + FC + master modality	D4 - CW	16
• WP2.b	Cyclic nested sequents for intuitionistic *Propositional Dynamic Logic*	D5 - JP	20
• WP3.a	Cyclic labelled sequents for the *Logic of Doxastic Upgrades* + RM	D6 - CW	24
• WP3.b	Cyclic labelled sequents for the *Epistemic Logic of TL Influence* + RM	D7 - PP	24

Figure 2: **Above:** Gantt chart summarising the work plan of CYDER. **Below:** tasks and associated deliverables. FC stands for *Frame Conditions* and RM for *Recursive Modalities*. PP stands for *Peer-reviewed Proceedings*, CW for *Communication to a Workshop* and JP for *Jounral Publication*. The two milestones coincide with WP1.b and WP2.a.

are expected in computer science (formal verification and knowledge representation), mathematics and philosophy (treatment of induction). By defining cyclic proofs based on nested and labelled proof systems for modal fixpoint logics, CYDER will widen the scope of existing proof-theoretical methods for modal logics. CYDER methodology can also be applied to other logics, offering a powerful tool to treat recurive modalities within, e.g., substructural and non-normal modal logics. The first proof-theoretical analysis of IML and DEL extended with recursive modalities proposed by CYDER will offer the possibility to prove additional result for these logics, such as decidability and interpolation. CYDER will contribute to gain a deeper understanding of cyclic proofs and their relationship with (co-)inductive reasoning.

Expected technological impact CYDER research has a foundational and theoretical nature, with important implications in **formal verification**, the scientific approach to software verification and certification. Temporal, dynamic and intuitionistic modal logics are used to express properties of programs, e.g. *safety* and *liveliness*. Accordingly, the compliance of a specific software to the property of interest can be verified by checking the satisfiablity of the corresponding formula. By offering a deep understanding of the logics involved, as well as a proof-theoretical framework general enough to cover several case-studies with the same methodology, CYDER will offer a timely and significant contribution in this field, potentially allowing to save precious time and resources in the lengthy and complex process of software verification.

Expected societal impact DEL with recursive modalities are a powerful tool to formalise epistemic phenomena such as common knowledge, the effects of learning new information, and the spread of belief through social networks. CYDER will offer a proof-theoretical analysis of these logics, allowing to gain a deep mathematical understanding of the social phenomena they model and, consequently, of our society. CYDER will actively engage in spreading basic knowledge about the logical mechanism underlying social communication, spreading awareness about the threats posed by the newly introduced social communication channels, e.g. social networks.

3 Quality and Efficiency of the Implementation

3.1 Quality and effectiveness of the work plan, assessment of risks and appropriateness of the effort assigned to work packages

CYDER is organised in three work packages (Fig. 2). WP1 addresses the core methodology of CYDER and, specifically, will introduce the basic strategies to define cyclic proof systems for modal fixpoint logics with frame conditions. WP2 and WP3 extend the developed strategy to IML and DEL, respectively. The various **tasks** of the work packages and their interconnections are illustrated in Fig 2, that also lists **deliverables** and milestones. While interconnected, as relying on the same core methodology, the tasks can be developed independently, as long as they refer to different logical systems. Therefore, even failure of all the tasks of WP1, which is extremely unlikely to happen, would not jeopardise WP2 and WP3. The two **milestones** are located at the completion of WP1.b and WP2.a, marking the definition of the first cyclic system for a multi-modal logic with frame conditions and the first proof system for IML with recursive modalities. At each milestone, I will assess the progress made, and adapt the work plan as needed. The overall work plan is ambitious, but feasible in view of my past research activity and productivity.

Events I will (co-)organise a dissemination workshop at UvA (June 2023). In summer 2024 I will offer training for Master and Ph.D. students at the ESSLLI summer school. I will (co-)organise and participate to the frequent outreach activities at UvA (Sec. 2.2) throughout the whole duration of the action.

Risk assessment As with any ambitious project, CYDER entails risks. Main risks and relevant mitigation strategies are listed below. Risks of physical nature, e.g. linked to fire, are unlikely to happen, and will be mitigated by storing drafts and working documents in remote repositories.

Risk	Mitigation action	Severity
Some modal fixpoint logics with frame conditions are highly undecidable.	I will rely on the model-theoretic expertise of Prof. Venema and, if needed, I will consider weaker frame conditions.	medium
Difficulties in defining proof systems for the alternation-free modal μ-calculus	In case of difficulties, I will consider smaller fragments of the modal μ-calculus, e.g. the linear-time μ-calculus.	high
Difficulties in defining cyclic proof for IML with recursive modalities	If needed, I will define infinitary proof systems with ω-rules instead of cyclic proofs.	medium high
Difficulties in defining rules for dynamic operators in DEL	In case of problems, I will define simple labelled rules inspired by the axioms of dynamic modalities.	low medium

3.2 Quality and capacity of the host institutions and participating organisations, including hosting arrangements

I will integrate the ILLC, a word-renowned logic institute, whose tradition goes back to Brouwer, Heyting, Beth and Troelstra. Nowadays, outstanding logicians such as de Jongh and van Benthem, figure among its members. ILLC is a lively and bustling center for studying modal logics, epistemic logics and algebraic semantics, and offers a highly-qualified Master training program in logic. During the action, I will interact on a weekly basis with Prof. Venema and Dr. Afshari, supervisors of CYDER, and will have frequent interactions with Prof. Smets and Dr. Baltag, experts of epistemic logics and topological semantics. More generally, I will be part of a stimulating research environment. My scientific integration within the ILLC will be ensured by my regular participation to research seminars[37], both as a speaker and as an attendee, by my proposal of Master projects, and by my involvement in outreach activities. CYDER will be granted qualified working space and full access to the UvA and ILLC technical resources.

The Human Resources office of the UvA, as well as the ILLC office, will offer support to the project on administrative and financial matters, as well as on other practical arrangements that might be required.

[37] https://events.illc.uva.nl/alg-coalg/